Review of Binomial Distribution

Let's recall that a **binomial experiment** consists of a fixed number of trials n. Each trial has exactly two possible outcomes (usually written as success(S)/failure(F) or Head(H)/Tail(T)). The probability of success, p, is the same on every trial. The trials are independent of one another. The probability of getting x successes ($0 \leq x \leq n$) is given by the following formula.

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

We say x is a binomial variable. For a binomial variable x, its mean value is $\mu = np$, and it's variance value is $\sigma^2 = np(1-p)$. To get the standard deviation of x, we take the square root of its variance.

1. A quiz consists of 10 true or false questions. To pass the quiz a student must answer at least eight questions correctly. If the student guesses on each question, what is the probability that the student will pass the quiz? Hint: We want to find $P(8) + P(9) + P(10)$.

A) 0.8 B) 0.08 C) 0.20 D) 0.055

Normal Distribution

2. Suppose X and Y are normal random variables with possibly different mean and standard deviation. Which of the following statements is **false**? (Hint: The total area under any normal curve is always 1. A larger σ produces a wider curve, so the peak must be lower to keep the area equal to 1.)

A) If the diagram of X has a higher peak than the diagram of Y, then $\sigma_X < \sigma_Y$.

B) If the diagrams of X and Y have the same peak height, then $\sigma_X = \sigma_Y$.

C) If the diagrams of X and Y have the same shape but the diagram of X is shifted to the right of Y, then $\mu_X > \mu_Y$.

D) If the diagram of X has a higher peak than the diagram of Y, then $\mu_X > \mu_Y$.

3. Find the value of $z_{0.05}$.

A) 1.75 B) 1.645 C) 0.52 D) -1.645

4. True or False: $z_{0.1}$ is the area under the standard normal curve to the right of $z = 0.1$.

A) True B) False

5. True or False: $P(z \geq 0.1)$ is the area under the standard normal curve to the right of $z = 0.1$.

A) True B) False

6. A physical fitness association is including the mile run in its high school fitness test. The time for this event is known to possess a normal distribution with a mean of 470 seconds

and a standard deviation of 50 seconds. Find the probability that a randomly selected high school student can run the mile in less than 355 seconds.

A) 0.5107 B) 0.0107 C) 0.9893 D) 0.4893

7. Assume that the random variable X is normally distributed, with mean $\mu = 80$ and standard deviation $\sigma = 16$. Compute the probability $P(36 < X < 100)$.

A) 0.8944 B) 0.7888 C) 0.8914 D) 0.8819

Using the Normal Distribution to Approximate a Binomial Distribution

8. True or False: In order to use a normal approximation to the binomial probability distribution, $np(1-p) \geq 10$.

A) True B) False

9. True or False: it is appropriate to use the normal distribution to approximate a binomial distribution with $n = 11$ and $p = 0.4$.

A) True B) False

10. Assuming that all conditions are met to approximate a binomial probability distribution with the standard normal distribution, then to compute $P(12 \leq x \leq 15)$ from the binomial distribution we must compute _____ as the normal approximation.

A) $P(12.5 < x < 14.5)$

B) $P(11.5 < x < 15.5)$

C) $P(x > 15.5)$ and $P(x < 11.5)$

D) $P(x > 12.5)$ and $P(x < 14.5)$

11. Assuming that all conditions are met to approximate a binomial probability distribution with the standard normal distribution, then to compute $P(x \geq 19)$ from the binomial distribution we must compute _____ as the normal approximation.

A) $P(x \geq 19.1)$ B) $P(x \leq 18.9)$ C) $P(x \geq 18.5)$ D) $P(x \leq 18.5)$

12. Find the probability that in 200 tosses of a fair six-sided die, a three will be obtained at least 40 times. (Hint: We can realize this experiment as a binomial experiment by setting the possible outcomes of a trial to be S: get a three; F: get any other number.)

A) 0.1190 B) 0.3875 C) 0.0871 D) 0.8810

Solution. 1. D. We plug $p = 0.5$, $n = 10$ into the binomial probability formula to calculate the cases of $x = 8, 9, 10$. Then we find the sum of $P(8), P(9), P(10)$.

2. D. Change of μ shifts the curve horizontally, but does not affect the shape of the normal curve.

3. B

4. B. $z_{0.1}$ is a possible value that the standard normal variable z can assume. On the diagram of the standard normal curve, $z_{0.1}$ is a point on the horizontal axis. $z_{0.1}$ is not an area.

5. A.

6. B. Let $z = \dfrac{x - 470}{50}$.

$$P(x < 355) = P(\frac{x - 470}{50} < \frac{355 - 470}{50}) = P(z < -2.3) = 0.0107.$$

7. C.

8. A.

9. B

10. B.

11. C.

12. A.