MA 116 B1 Assignment 1

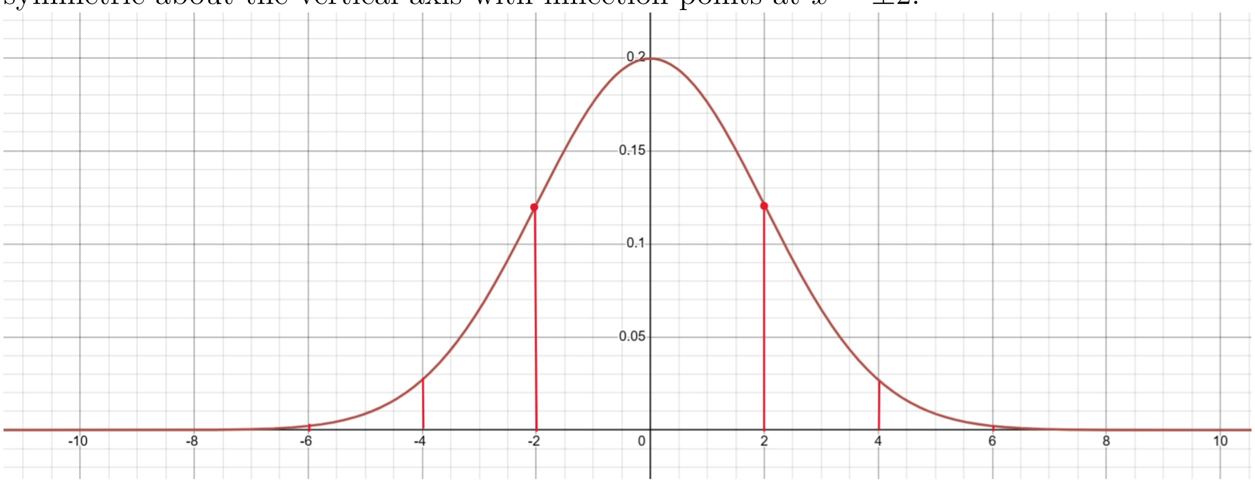
Due July 04 11:59 pm

Question 1

The graph below shows part of a Bell curve

$$f(x) = \frac{1}{2\sqrt{2\pi}} \exp\left(\frac{-1}{2}\left[\frac{x}{2}\right]^2\right)$$

symmetric about the vertical axis with inflection points at $x = \pm 2$.



(a) Is $f(x)$ a standard normal curve? Provide an answer along with a brief explanation of your reasoning.

(b) Explain why $f(x)$ is a good probability density function.

- What is the total area between the curve of $f(x)$ and the x-axis?
- Does there exist an x value at which $f(x) < 0$?

(c) What is the area between the curve of $f(x)$ and the x-axis in the region $x > 0$? Explain your answer.

(d) By the empirical rule, what is the area between the curve of $f(x)$ and the x-axis in the region $-2 \leq x \leq 2$? What is the area between the curvve of $f(x)$ and the x-axis in the region $x > 4$?

Question 2

In the US, human adult height is approximately normally distributed with a mean of 170 cm and a standard deviation of 9 cm. What is the probability that a randomly chosed US adult has a height ≥ 174.5 cm? Show your steps.

Question 3

Suppose the measurement (in nanometers, may be a positive, negative, or zero value) of a physics experiment measuring the relative position of a particle can be approximated by a **standard** normal distribution. Provide answers along with brief explanations of your reasoning for the following questions.

(a) What is the probability that the result of one experiment is larger than 0?

(b) What is the probability that the result of one experiment is between -0.4 and 0.7?

(c) Given $z_{0.025} = 1.96$, what is the probability that the result of one experiment is larger than 1.96?

(d) Find $z_{0.03}$.

(e) Prove the lemma $-z_\alpha = z_{1-\alpha}$.

Question 4

Suppose a function F defined on the domain $-2 \leq x \leq 2$ is a uniform probability density function.

(a) Draw a diagram for $F(x)$.

(b) What is the value of $F(x)$ for any $-2 \leq x \leq 2$? Label the value on the vertical axis of your diagram in (a).

(c) Find $P(-1 \leq x \leq 0.3)$.

(d) Suppose a is a number between -2 and 2 such that the area between the curve of F and the horizontal axis to the right of a is 0.1. I.e. $P(x > a) = 0.1$. Find the value of a.

(e) Define a new function $D(x) = F(x) - 1$. Explain why or why not $D(x)$ is a good probability density function.

Question 5

Consider a quantitative sample given below.

Index (i)	x
1	0.3
2	-4
3	-0.9
4	1
5	0

(a) Calculate

$$\sum_{i=1}^{5} x_i.$$

(b) Calculate the sample mean $\overline{x}$ of this sample.

(c) Calculate the sample standard deviation s of this sample.

Question 6

Recall from MA 115 that the probability distribution of a binomial random variable x is given by

$$\binom{n}{x} p^x q^{n-x},$$

where n is the total number of trials, p is the probability of getting S (success) in one single trial, and $q = 1 - p$ is the probability of getting F (fail) in one single trial.

(a) State the experiment associated to this binomial random variable x.

(b) What is the sample space of this variable x?

(c) Given $n = 3$, $p = 0.3$, verify that the probability of x being equal to each possible sample point sums to 1.

(d) Let's realize a trial as an experiment. Let y be the variable associated to the outcome of this experiment. Describe the sample space and probability distribution of this variable y.

Formula Sheet

Population mean formula. $\mu = \frac{\sum x_i}{N}$ where the summation is taken over all data points in the population, and N is the population size.

Population variance formula. $\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$ where the summation is taken over all data points in the population, and N is the population size.

Population standard deviation formula. $\sigma = \sqrt{\sigma^2}$.

Sample mean formula. $\overline{x} = \frac{\sum_{i=1}^n x_i}{n}$, where n is the sample size.

Sample variance formula. $s^2 = \frac{\sum_{i=1}^n (x_i - \overline{x})^2}{n-1}$, where n is the sample size.

Sample standard deviation formula. $s = \sqrt{s^2}$.

Normal distrubution/Bell curve

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-1/2[(x-\mu)/\sigma]^2)$$

Change of Variable formulas.

Given a normal variable x with a mean μ and a standard deviation σ, we may convert it to a standard normal variable z by the formula

$$z = \frac{x-\mu}{\sigma}$$

Then,

$$P(\frac{x-\mu}{\sigma} \leq a) = P(x \leq a\sigma + \mu)$$

$$P(\frac{x-\mu}{\sigma} \geq a) = P(x \geq a\sigma + \mu)$$

for any number a.

$$P(x \leq a) = P(z \leq \frac{a-\mu}{\sigma})$$

$$P(x \geq a) = P(z \geq \frac{a-\mu}{\sigma})$$

for any number a.

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z_α formulas.

- $P(z \geq z_\alpha) = \alpha$.
- For any $0 \leq \alpha \leq 1$ we have $-z_\alpha = z_{1-\alpha}$.

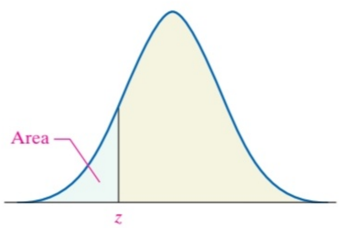

Table V

Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

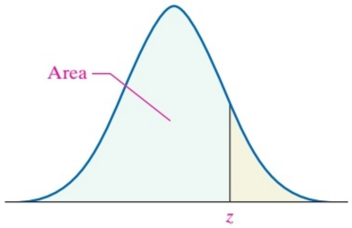

Table V (*continued*)

Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Reference of the standard normal distribution table. *STATISTICS: Informed Decisions Using Data*, Michael Sullivan, 7e.