1 Hypothesis test: classical approach

1.1 Calculating the test statistics

The idea of hypothesis test (classical approach) is to first assume that H_0 holds to get a hypothetical distribution of the variable (let's use $\overline{x}$) that we are concering about. If a random sample turns out to have a sample statistic $\overline{x}$ that lies near the peak of the hypothetical distribution, then roughly speaking, this sample agrees with H_0. If a random sample turns out to have a sample statistic $\overline{x}$ that lies in the left and/or right tail(s) (depend on test type), then roughly speaking, under the assumption that H_0 holds, obtaining a random sample like this is unlikely, so we want to use this sample as evidence to reject H_0.

We make the critical regions rigorous by changing to the distribution of a new variable t, so that we can say critical values are t_α, $-t_\alpha, \pm t_{\alpha/2}$ (depending on test type). The distribution of the new variable t is determined under the assumption that $H_0 : \mu = \mu_0$ holds, so $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$. In this formula, μ_0 and n are numerical values, while $\overline{x}$, s varies with choice of random sample. To calculate t_0, we first calculate the sample statistics $\overline{x}$ and s from our particular sample, and plug these two numerical values into $\dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$ to get t_0.

In Quiz 2 Question 5, when calculating the test statistic z_0, some students used

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where they take $\hat{p}$ to be the sample statistics of our particumar sample. This is not accurate. The correct formula to use is $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$, where p is a numerical value obtained from assuming H_0 holds. There are two things to notice

1. The distribution of the new variable z is determined under the assumption that H_0 holds
2. Even though we have sometimes used $\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ to appriximate $\sqrt{\dfrac{p(1-p)}{n}}$, we should not use the approximating formula here because we know the value of p from assuming H_0 is true.

Another thing to notice in calculating t_0 is the formula for the sample standard deviation s, which is $\sqrt{\dfrac{\sum_i (x_i - \overline{x})^2}{n-1}}$. Example: my sample is $\{2, 3, 5\}$. Then the $\overline{x}$ of my sample is $\dfrac{2+3+5}{3} = 10/3 \approx 3.3$. We calculate s by

$$s = \sqrt{\frac{\sum_i (x_i - 3.3)^2}{n-1}} = ?$$

1.2 Rubric for hypothesis test (classical approach) questions

1. Determine H_0, H_1, test type (2)
2. Verify that the conditions for our variable to have a desired distribution are satisfied. Eg. Variable is $\overline{x}$, so we need to varify that either the original population is normally distributed, or $n > 30$, in order for the new variable $t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$ to follow Student's t-distribution of df=n-1. (2) In Quiz 2 Question 5, some students forgot to verify if $np(1-p) \geq 10$.
3. Determine the critical value(s). (2)
4. Calculate the test statistic and check if the test statistic falls in the critical region. (2)
5. Draw conclusion to your hypothesis test. (2) It is wrong to say that there is sufficient evidence to **accept** H_0! You must say **There is/is not sufficient evidence to conclude [H_1 statement]**.

Exercise. Find $t_{0.01}$ with $df = 20$.

2 Quiz 3

2.1 Structure

1. Question 1. Hypothesis test about a population mean, classical approach
2. Question 2. Hypothesis test about two population proportion–independent sample
3. Question 3. Hypothesis test about two population mean–matched pair data
4. Question 4. Hypothesis test about two mean–independent sample

2.2 Practice question 1. Hypothesis test about a population mean, classical approach

The average phone screen time of Americans is about 5 hours. We want to know if Americans aged 60 and older has a lower average phone screen time. Suppose we have a random sample of size 36 from Americans aged 60 and older with a sample mean $\overline{x} = 4.1$ and a sample variance $s = 2$. Conduct a hypothesis test with $\alpha = 0.02$.

2.3 Practice question 2. Hypothesis test about two population proportions, independent samples

A pharmaceutical had developed an updated version of an existing vaccine and wanted to know if the new version offers a higher rate of protection against flu compared to the older version. Researchers conducted two independent randomized trials:

1. In the first trial, 400 individuals received the updated vaccine, and 340 of them did not contract the flu.

2. In the second trial, 400 individuals received the older version of vaccine, and 330 of them did not contract the flu.

Conduct a hypothesis test at $\alpha = 0.05$ significance.

2.4 Practice question 3. Hypothesis test about two population means, matched pair data

A company wants to test whether a new training program improves employee productivity. To investigate this, a random sample of 5 employees is selected. For each employee, the number of units produced per day is recorded **before** the training and again **after** the training. Let x_i be the number of units produced **before** training, and y_i be the number of units produced **after** training for employee i.

1. A random sample is $\{(10, 10.5), (8, 11.2), (3.2, 8.4), (7.5, 15), (15, 24.6)\}$. Calculate $\overline{d}$ and s_d of this sample.

2. Assume d is normally distributed. Conduct a hypothesis test at the $\alpha = 0.05$ significance level.

2.5 Practice question 4. Hypothesis test about two population means, independent samples

RMK. Two cases that the new variable t follows Student's t-distribution with the smaller of $n_1 - 1$, $n_2 - 1$ df: 1. Both populations are normal; 2. Both n_1, $n_2 > 30$.

We want to know if an experimental drug relieves symptoms attributable to the common cold. Let μ_1 be the mean time until cold go away for anyone who (hypothetically) take the drug. Let μ_2 be the mean time until cold go away for anyone who is not taking this drug. We assume x_1 and x_2 are approximately normal variables.

- 14 individuals with colds are randomly assigned to take the drug (Group 1).
- Another 14 individuals are randomly assigned to take a placebo (Group 2).

Group 1 (Drug): sample mean $\overline{x}_1 = 5.9$ days, sample standard deviation $s_1 = 1.2$ days.
Group 2 (Placebo): sample mean $\overline{x}_2 = 7.1$ days, sample standard deviation $s_2 = 1.5$ days.

Do a hypothesis test at $\alpha = 0.05$.
